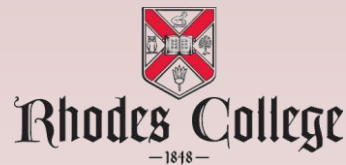


COMP 355

Advanced Algorithms

Divide and Conquer: Selection
KT: 5.4



Selection Algorithm

Selection by the Sieve Technique

```
Select(array A, int p, int r, int k) { // return kth smallest of A[p..r]
  if (p == r) return A[p]           // only 1 item left, return it
  else {
    x = ChoosePivot(A, p, r)         // choose the pivot element
    q = Partition(A, p, r, x)        // <A[p..q-1], x, A[q+1..r]>
    xRank = q - p + 1                // rank of the pivot
    if (k == xRank) return x         // the pivot is the kth smallest
    else if (k < xRank)
      return Select(A, p, q-1, k)    // select from left
    else
      return Select(A, q+1, r, k-xRank)// select from right
  }
}
```

Selection Algorithm

Lemma: The element x is of rank at least $n/4$ and at most $3n/4$ in A .

14	32	23	5	10	60	29	6	2	3	5	8	1	11
57	2	52	44	27	21	11	14	25	12	17	10	21	29
24	43	12	17	48	1	58	24	30	23	34	19	41	39
6	30	63	34	8	55	39	37	32	52	44	27	55	58
37	25	3	64	19	41		57	43	63	64	48	60	

Group

Get group medians

8	3	6	2	5	11	1
10	12	14	25	17	29	21
19	23	24	30	34	39	41
27	52	37	32	44	58	55
48	63	57	43	64		60

Get median of medians

(Sorting of group medians is not really performed)

Analysis of Choose Pivot

$$T(n) \leq \begin{cases} 1 & \text{if } n = 1, \\ T(n/5) + T(3n/4) + n & \text{otherwise.} \end{cases}$$

Theorem: There is a constant c , such that $T(n) \leq cn$.

Proof: (by strong induction on n)

Basis: ($n = 1$) In this case we have $T(n) = 1$, and so $T(n) \leq cn$ as long as $c \geq 1$.

Step: We assume that $T(n') \leq cn'$ for all $n' < n$. We will then show that $T(n) \leq cn$.

By definition we have

$$T(n) = T(n/5) + T(3n/4) + n.$$

Since $n/5$ and $3n/4$ are both less than n , we can apply the induction hypothesis, giving

$$\begin{aligned} T(n) &\leq c\frac{n}{5} + c\frac{3n}{4} + n = cn \left(\frac{1}{5} + \frac{3}{4} \right) + n \\ &= cn\frac{19}{20} + n = n \left(\frac{19c}{20} + 1 \right). \end{aligned}$$

This last expression will be $\leq cn$, provided that we select c such that $c \geq (19c/20)+1$.

Solving for c we see that this is true provided that $c \geq 20$.

Combining the constraints that $c \geq 1$, and $c \geq 20$, we see that by letting $c = 20$, we are done.