COMP 355
Advanced Algorithms

Divide and Conquer: Selection
KT: 5.4

Rhodes College
—1848—

Selection Algorithm


```
Select(array A, int p, int r, int k) {  // return kth smallest of A[p..r]
    if (p == r) return A[p]             // only 1 item left, return it
    else {
        x = ChoosePivot(A, p, r)        // choose the pivot element
        q = Partition(A, p, r, x)       // <A[p..q-1], x, A[q+1..r]>
        xRank = q - p + 1               // rank of the pivot
        if (k == xRank) return x        // the pivot is the kth smallest
        else if (k < xRank)
            return Select(A, p, q-1, k) // select from left
        else
            return Select(A, q+1, r, k-xRank)// select from right
    }
}
```

Selection Algorithm

Lemma: The element x is of rank at least $n/4$ and at most $3n/4$ in A.

14	32	23	5	10	60	29
57	2	52	44	27	21	11
24	43	12	17	48	1	58
6	30	63	34	8	55	39
37	25	3	64	19	41	

Group

6	2	3	5	8	1	11
14	25	12	17	10	21	29
24	30	23	34	19	41	39
37	32	52	44	27	55	58
57	43	63	64	48	60	

Get group medians

8	3	6	2	5	11	1
10	12	14	25	17	29	21
19	23	24	30	34	39	41
27	52	37	32	44	58	55
48	63	57	43	64		60

Get median of medians
(Sorting of group medians is not really performed)

Analysis of Choose Pivot

$$T(n) \le \begin{cases} 1 & \text{if } n = 1, \\ T(n/5) + T(3n/4) + n & \text{otherwise.} \end{cases}$$

Theorem: There is a constant c, such that $T(n) \le cn$.

Proof: (by strong induction on n)

Basis: ($n = 1$) In this case we have $T(n) = 1$, and so $T(n) \le cn$ as long as $c \ge 1$.

Step: We assume that $T(n') \le cn'$ for all $n' < n$. We will then show that $T(n) \le cn$. By definition we have

$$T(n) = T(n/5) + T(3n/4) + n.$$

Since $n/5$ and $3n/4$ are both less than n, we can apply the induction hypothesis, giving

$$\begin{aligned} T(n) &\le c\frac{n}{5} + c\frac{3n}{4} + n = cn\left(\frac{1}{5} + \frac{3}{4}\right) + n \\ &= cn\frac{19}{20} + n = n\left(\frac{19c}{20} + 1\right). \end{aligned}$$

This last expression will be $\le cn$, provided that we select c such that $c \ge (19c/20)+1$. Solving for c we see that this is true provided that $c \ge 20$.

Combining the constraints that $c \ge 1$, and $c \ge 20$, we see that by letting $c = 20$, we are done.