Problem Set 11: Approximation Algorithms

Handed out Friday, November 15. Due at the start of class Friday, November 22.

Problem 1.(15 points) Given a graph $G = (V, E)$, a subset of vertices $V' \subseteq V$ is called a *dominating set* if every vertex of G is either in the set V' or is a neighbor of a vertex in V'. For example, in Fig. 1(a) the set $\{4, 8\}$ is a dominating set of size two. In the optimization version of the *dominating set problem* you are given a graph G and the objective is to compute a dominating set of minimum size.

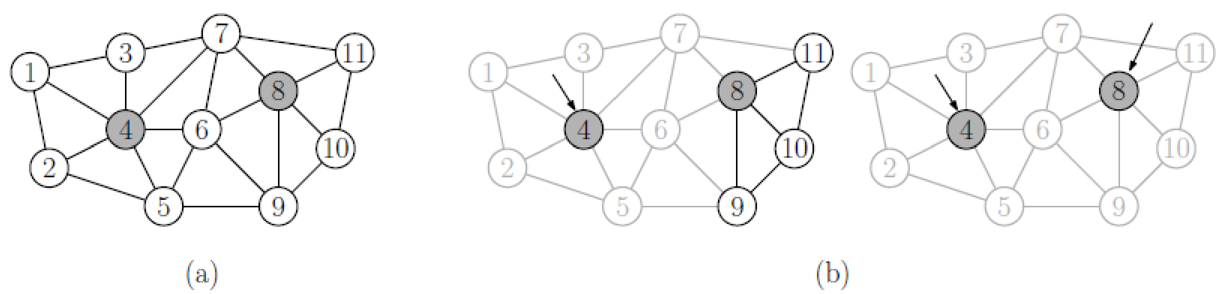

Figure 1: Problem 1: (a) Dominating set and (b) the greedy heuristic.

In class, we showed that the *dominating set problem* is NP-Complete. However, we would like to find an approximate solution to this problem.

Consider the following greedy heuristic for this problem. Initially, all vertices are marks as undominated. Select a vertex that is adjacent to the maximum number of undominated vertex. Add this vertex to the dominating set. Mark it and all its neighbors as dominated. Repeat until no more undominated vertices remain. (For example, in Fig. 1(b) the heuristic would first select 4 which is adjacent to six undominated vertices, and then it could select either vertex 8 or 10, each of which is adjacent to three undominated vertices.)

(a) Present an example to show that this greedy heuristic is not optimal.

(b) Show that the greedy heuristic achieves an approximation factor of $\ln n$, where $n = |V|$. That is, if there exists a dominating set of size k, then the greedy heuristic outputs a dominating set of size at most $k \cdot \ln n$.

Problem 2.(15 points) You are working for a private space corporation that wants to configure its next space mission. The job is to fill a rocket with a set of scientific experiments to be run in space. There are n experiments that have been proposed as candidates to be on the mission, but their total weight is more than the rocket can lift. You have been asked to determine the best subset of experiments to launch on the mission.

For $1 \leq i \leq n$, let w_i denote the weight of the ith experiment. Let W denote the total weight that can be carried by the rocket. The objective is to determine the subset of experiments whose total weight comes as close to W without exceeding this value.

More formally, given the weights $\langle w_1, ..., w_n \rangle$ and you want to compute the subset $E \subset \{1, ..., n\}$, to maximize $\sum_{i \in E} w_i$ subject to the constraint $\sum_{i \in E} w_i \leq W$.

(a) Suggest a greedy approach for solving this problem. That is, you will sort the items according to some statistic, and then take as many items as possible (according to your ordering) as long as the total weight does not exceed W. How would you order the experiments? (No explanation is required. But please read (b) before trying to prove that your algorithm is optimal!)

(b) Show that your greedy algorithm is *not* optimal by showing that there is a set of weights such that your algorithm fails to achieve the optimum.

(c) Show that your greedy algorithm is not that bad after all, by proving that if the optimum algorithm achieves a total weight of W_O, your greedy algorithm will achieve a total weight of $W_G \geq W_O/2$.