Network Flow Practice

Problem 1. Your friend has a new drone delivery startup, and he has asked you to help him by designing software to assist with scheduling deliveries.

- There are m drone stations throughout the city. For $1 \leq i \leq m$, let $d_i = (d_{i,x}, d_{i,y})$ denote the (x, y) coordinates of the ith drone station (see Fig. 1(a)). Due to FAA regulations, each drone station can launch no more than *5 drones* each day.
- There are n customers expecting to receive a package this day. For $1 \leq j \leq n$, let $c_i = (c_{j,x}, c_{j,y})$ denote the (x, y) coordinates of the jth customer. (You may assume that no two customers occupy the same location, and each customer is expected exactly one delivery.)
- Each drone station is attached to complete warehouse, so in theory a drone from any station can deliver the desired package to any customer. However, because of fuel limitations, each drone can make a delivery only within a 10 mile radius of the station (see Fig. 1(a)). That is, station i can only deliver packages to those customers j such that $\text{dist}(d_i, c_j) \leq 10$.

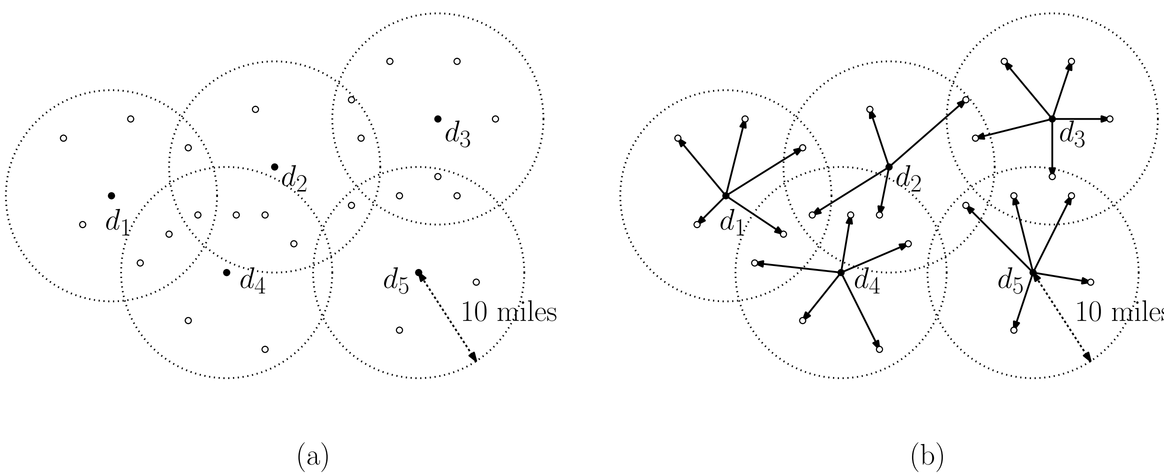

Figure 1: Drone delivery service. Black points are drone stations and hollow points are customers: (a) Input and (b) Possible solution.

Your algorithm is given the coordinates of the m drone stations and the coordinates of the n customers. The problem is to determine the maximum number of deliveries that can be made (ideally all n of them), subject to the constraints given above (see Fig. 1(b)).

(Hint: Reduce to network flow. Give both the reduction and a proof that your reduction is correct. Given the flow output, explain how to determine the set of customers that station i will ship to.)

Problem 2. Professor Adam has two children, who unfortunately, dislike each other. The problem is so severe that not only do they refuse to walk to school together, but in fact each one refuses to walk on any block that the other child has stepped on that day. The children have no problem with their paths crossing at a corner. Fortunately, both the professor's house and the school are on corners, but beyond that he is not sure if it is going to be possible to send both of his children to the same school. The professor has a map of the town. Show how to formulate the problem of determining whether both his children can go to the same school as a maximum-flow problem.