COMP 355 Advanced Algorithms

Divide and Conquer: Selection KT: 5.4



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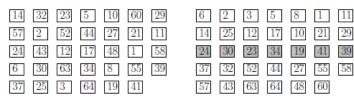
Selection Algorithm



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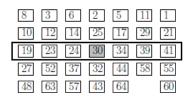
Selection Algorithm

Lemma: The element x is of rank at least n/4 and at most 3n/4 in A.



Group

Get group medians



Get median of medians (Sorting of group medians is not really performed)

Analysis of Choose Pivot

$$T(n) \leq \left\{ \begin{array}{ll} 1 & \text{if } n=1, \\ T(n/5) + T(3n/4) + n & \text{otherwise}. \end{array} \right.$$

Theorem: There is a constant c, such that $T(n) \leq cn$.

Proof: (by strong induction on n)

Basis: (n=1) In this case we have T(n)=1, and so $T(n) \le cn$ as long as $c \ge 1$. Step: We assume that $T(n') \le cn'$ for all n' < n. We will then show that $T(n) \le cn$. By definition we have

$$T(n) = T(n/5) + T(3n/4) + n.$$

Since n/5 and 3n/4 are both less than n, we can apply the induction hypothesis, giving

$$\begin{split} T(n) & \leq & c\frac{n}{5} + c\frac{3n}{4} + n = & cn\left(\frac{1}{5} + \frac{3}{4}\right) + n \\ & = & cn\frac{19}{20} + n = & n\left(\frac{19c}{20} + 1\right). \end{split}$$

This last expression will be $\leq cn$, provided that we select c such that $c \geq (19c/20) + 1$. Solving for c we see that this is true provided that $c \geq 20$.

Combining the constraints that $c \ge 1$, and $c \ge 20$, we see that by letting c = 20, we are done.

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