CS 355: Advanced Algorithms **Fall 2017**

NP-Complete Practice Problems

1. The Conflict Set Problem (CONF) is as follows. The input is a pair (S, k) consisting of a collection of sets S = $\{S_1, \ldots, S_n\}$ over some finite domain, and a positive integer k. The question is whether there exists a set C of size k such that every set of S has a nonempty intersection with C. That is, whether for all $1 \leq i \leq n$, $S_i \cap C \neq \emptyset$. (We say that C conflicts with S_i.) For example, let $S_1 = \{1, 2, 3\}$, $S_2 = \{1, 4, 5\}$, $S_3 = \{2, 4, 6\}$, $S_4 = \{2, 5, 7\}$, $S_5 = \{3, 7, 9\}$, and let $S = \{S_1, \ldots, S_5\}$. There exists a conflict set of size 3 consisting of C = {2, 5, 7}. Therefore (S, 3) ∈ CONF. However, there is no conflict set of size 2 for S (since no matter which two elements you pick, some set will fail to contain at least one of them), and therefore (S, 2) does not exist in CONF.

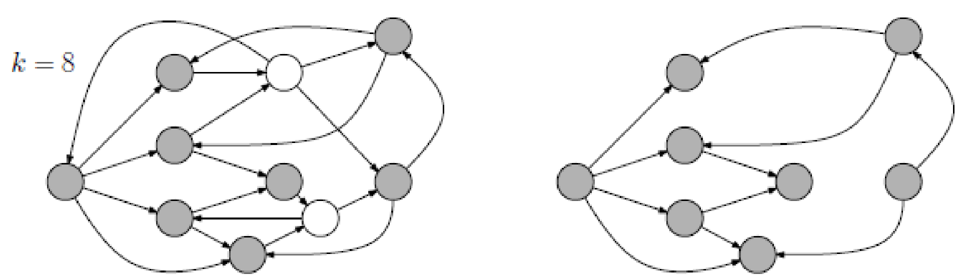
   The goal of this problem is to show that CONF is NP-Complete.

   a. Briefly explain why CONF is in NP.
   b. Prove that CONF is NP-hard by showing that the vertex cover problem, VC, is polynomially reducible to CONF.

2. Given an undirected graph G = (V, E) and a subset V′ ⊆ V, the induced subgraph on V′ is the subgraph G′ = (V′, E′) whose vertex set is V′, and for which (u, v) ∈ E′ if u, v ∈ V′ and (u, v) ∈ E. The acyclic subgraph problem (AS) is as follows. Given a directed graph G = (V, E) and an integer k, does G contain a subset V′ of k vertices such that the induced subgraph on V′ is acyclic? (For example, in Fig. 1(a), we show a graph that has an acyclic subgraph of size k = 8. I don't believe a larger acyclic subgraph exists. Fig. 1(b) shows the acyclic induced subgraph.)

   The goal of this problem is to show that AS is NP-Complete.

   a. Briefly explain why AS is in NP.
   b. Prove that AS is NP-hard by showing that the independent set problem, IS, is polynomially reducible to AS.



(a) **Figure 1: Problem 2** (b)