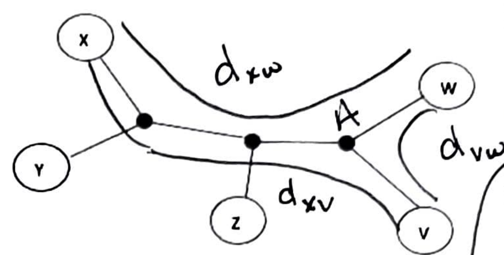
Small Additive Distance Phylogeny Problem:
Given: An $n \times n$ additive distance matrix D and an unweighted Phylogenetic tree T
Find: Branch lengths such that $d_T(i,j) = D_{ij}$ for all $1 \le i, j \le n$.

In this example, the Four Point Condition was used to reconstruct the branch lengths for the given unweighted Phylogenetic tree T. Follow through this example and make sure you understand how to get the solution. Then try it yourself on the Practice Problem on Page 2.

D

	V	W	X	Y	Z
V	0	10	17	16	16
W	-	0	15	14	14
X	-	-	0	9	15
Y	-	-	-	0	14
Z	-	-	-	-	0



Because D is additive

$d_{AX} + d_{AW} = d_{XW}$

$d_{AX} + d_{AV} = d_{XV}$

$d_{AW} + d_{AV} = d_{WV}$

Need to solve for unknowns. Add equations to do this.

$d_{AX} + d_{AW} = d_{XW}$

$+\ d_{AX} + d_{AV} = d_{XV}$

$2d_{AX} + \underbrace{d_{AW} + d_{AV}}_{d_{WV}} = d_{XW} + d_{XV}$

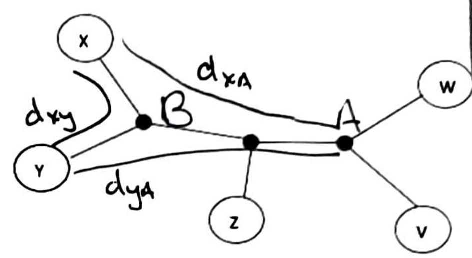
$2d_{AX} + d_{WV} = d_{XW} + d_{XV}$

$d_{AX} = \frac{1}{2}(d_{XW} + d_{XV} - d_{WV})$

Find neighbors V and W. Label common parent A.

see work on right for how to get this equation →

- $d_{AX} = \frac{1}{2}(d_{VX} + d_{WX} - d_{VW}) = 11$
- $d_{AY} = \frac{1}{2}(d_{VY} + d_{WY} - d_{VW}) = 10$
- $d_{AZ} = \frac{1}{2}(d_{VZ} + d_{WZ} - d_{VW}) = 10$

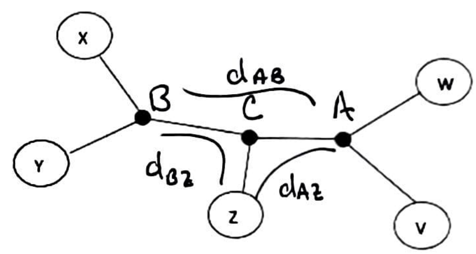


	A	X	Y	Z
A	0	11	10	10
X	-	0	9	15
Y	-	-	0	14
Z	-	-	-	0

removed W, V
added A

Find neighbors X and Y. Label common parent B.

- $d_{AB} = \frac{1}{2}(d_{AX} + d_{AY} - d_{XY}) = 6$
- $d_{BZ} = \frac{1}{2}(d_{XZ} + d_{YZ} - d_{XY}) = 10$

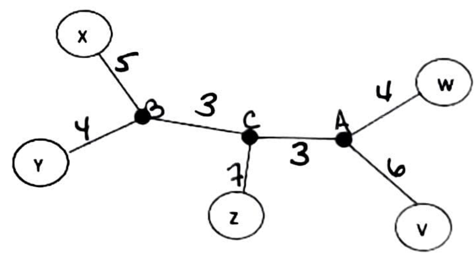


since $d_{AB} = 6$, and $d_{AC} = 3$, then $d_{BC} = 3$

	B	A	Z
B	0	6	10
A	-	0	10
Z	-	-	0

Find neighbors B and Z. Label common parent C.

- $d_{AC} = \frac{1}{2}(d_{AB} + d_{AZ} - d_{BZ}) = 3$



	C	A
C	0	3
A	-	0

Fill in distances going backwards.

start at d_{AC}, fill in.

Practice problem. Add the appropriate weights to the given Phylogenetic tree T for the additive distance matrix D.

D

	A	B	C	D	E
A	0	11	10	9	15
B	-	0	3	12	18
C	-	-	0	11	17
D	-	-	-	0	8
E	-	-	-	-	0

T