Discrete Structures, Spring 2013, Homework 9

You must write the solutions to these problems legibly on your own paper, with the problems in sequential order, and with all sheets stapled together.

For each of the following, give a proof of the statement if it is true, or a counterexample if the statement is false. Remember, counterexamples must include specific values and enough work shown to demonstrate that they are actual counterexamples.

1. For all sets A, B, and C, if $A \cup C = B \cup C$, then $A = B$.

2. For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \times B \subseteq B \times C$.

   Hint: Because we're proving a Cartesian product is a subset of another Cartesian product, after you begin with "Assume $A \subseteq B$ and $B \subseteq C$," you next write "Let (x, y) be an arbitrary element in $A \times B$." Normally this line would be "Let x be an arbitrary element in $A \times B$," but because we're dealing with Cartesian products, we need that second variable (y).

3. For all sets A and B, if $A \cap B = \emptyset$ then $A \times B = \emptyset$.

4. For all sets A, B, and C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.

5. For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

6. *(Save this one until after Monday's class. You can do it by proving subset in both directions like we've learned, but it's easier using a different method we'll see on Monday.)*

   Let $A_1, A_2, \ldots$ be any sets. Prove the following statement:

   $\forall n \in \mathbb{Z}^{\geq 2} \ (A_1 \cup A_2 \cup \cdots \cup A_n)^c = A_1^c \cap A_2^c \cap \cdots \cap A_n^c$.

   Hint: Use (regular) induction. You may also find these rules helpful:

   $A_1 \cup A_2 \cup \cdots \cup A_{k+1} = A_1 \cup A_2 \cup \cdots \cup A_k \cup A_{k+1}$
   $A_1 \cap A_2 \cap \cdots \cap A_{k+1} = A_1 \cap A_2 \cap \cdots \cap A_k \cap A_{k+1}$