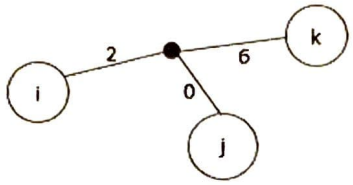
Large Additive Distance Phylogeny Problem:
Given: An additive $n \times n$ distance matrix D
Find: Phylogenetic T and branch lengths such that $d_T(i,j) = D_{ij}$ for all $1 \leq i, j \leq n$.

A **degenerate triple** is a set of three species i, j, k where $D_{ij} + D_{jk} = D_{ik}$.



$D_{ij} = 2$
$D_{ik} = 8$
$D_{jk} = 6$

Algorithm Idea:

- If D has a degenerate triple i, j, k, then j can be "removed" from D, reducing the size of the problem.
- Otherwise, you can create one by "shortening" all hanging edges in the tree by δ
- All paths between leaves then shrink by 2δ.
- Repeat until you have a 2×2 size matrix.
- "Traceback" through matrices, "re-grow" hanging edges, and insert removed nodes.

Work through this example to find the phylogenetic tree T and branch lengths.

	A	B	C	D
A	0	4	10	9
B	-	0	8	7
C	-	-	0	9
D	-	-	-	0

Subtract 2δ from all

$\delta = 1$

	A	B	C	D
A	0	2	8	7
B	-	0	6	5
C	-	-	0	7
D	-	-	-	0

Remove B

(Remember $D_{AB} = 2$ on path to C)

	A	C	D
A	0	8	7
C	-	0	7
D	-	-	0

trim

$\delta = 3$

	A	C	D
A	0	2	1
C	-	0	1
D	-	-	0

Remove D

(D_AD = 1 on path to C)

	A	C
A	0	
C	-	0

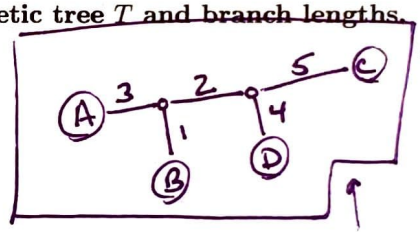
Degenerate Triple:
$i \leftarrow A, j \leftarrow B, k \leftarrow C$

$D_{AB} + D_{BC} = 2 + 6 = 8 = D_{AC}$

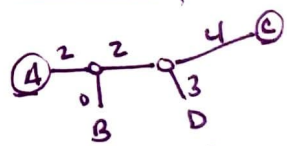
Degenerate Triple:
$i \leftarrow A, j \leftarrow D, k \leftarrow C$

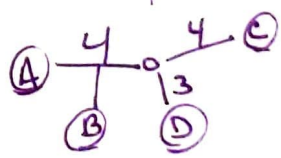
$D_{AD} + D_{DC} = 1 + 1 = 2 = D_{AC}$

final T



add 1 to hanging edges





Add 3 to hanging edges